(Short answer type questions) of B.Sc. Sem II physics (Hons.) Paper-4

Wave Motion.

Que1 What do you mean by 'wave motion'?

Ans: Wave motion is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean position, the motion being handed on from one particle to the next.

Que2: What do you mean by Transverse wave motion & Longitudinal wave motion?

Ans: The wave motion in which the particles of the medium vibrate about their mean position in a direction at right angle to the direction of propagation of the wave, then this motion is called Transverse wave motion.

But, the wave motion in which the particles of the medium vibrate about their mean position in the same direction in which the wave is propagated, then this type of wave motion is called Longitudinal wave motion.

Que3: What do you mean by Progressive wave & Stationary wave?

Ans: If a wave travels in a medium, the particles of the medium excutes simple harmonic vibration about their mean position, so a disturbance is produced. This disturbance is handed on from particle to particle after a definite time and there is a gradual fall of phase in the direction of motion, This disturbance which travels in a particular direction, is known as Progressive wave.

When the two waves which have equal wavelength, amplitude and frequency but opposite direction superimpose on each other give Stationary waves.

Que 4: Define the term wave velocity and group velocity

Ans: When a wave travels through a medium, the particles of the medium excute simple harmonic vibration about their mean position The individual oscillators which make the medium only excute and donot travel themselves through the medium with the wave. Every particle begins its vibration a little later than its predecessor and there is a progressive change of phase as we travel from one particle to the next. There are three distinct velocity connected with the wave motion.

(1) Particle velocity: It is the velocity of the simple harmonic motion of the oscillating particle about its equalibrium position

(2) Wave velocity or phase velocity: It is the velocity with which the plane of equal phase (crest or trough) travels through the medium.

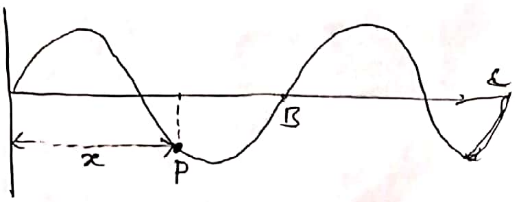
(3) The group velocity: A group consists of a no. of waves of different frequencies superimpose upon each other For example white light consists of a continous visible wavelength spectrum ranging from about 3000A° in the voilet to about 7000A° in the red region. The motion of such a pulse is described by its group velocity.

The importance of the group velocity lies in the fact that this is the velocity with which the energy in the wave group is transmitted. For a monochromatic wave the group velocity and wave velocity are identical

Que 5: Deduce the equation of wave motion of a progressive wave.

Ans: In a wave motion the particles of the medium excute simple Harmonic vibration about their mean position. These vibrating particle only excute and donot travel in the medium, but it is the wave which travel through the medium and this wave is called progressive wave.

Let us consider the wave is travelling from left to right, a particle on the right will begin its vibrations a certain time later than the one to its left.



As each particle of the medium excutes simple Harmonic motion the equation of motion of any particle A, is given by

$$y = a \sin \omega t$$

where a is the amplitude of vibrating particle, y is displacement after time t and ω is angular frequency, if n is the frequency then $\omega = 2\pi n$

So, $y = a \sin 2\pi n t$

when A passes through its mean position, the particles like B, C---etc also pass through their mean positions in the same directia.

Hence particle A, B, C are in the same phase

The distance between the two consecutive particles in the same phase is called wavelength λ. The change of phase from A to B is 2π

Therefore, in going from A to any point P at a distance x from A the phase changes by φ, so change of phase $\phi = \frac{2\pi}{\lambda} x$

Hence the displacement of P is given by, $y = a \sin(\omega t - \phi)$

$$y = a\sin\left(2\pi n t - \frac{2\pi}{\lambda}x\right)$$
$$= a\sin\left(2\pi \frac{v}{\lambda}t - \frac{2\pi}{\lambda}x\right)$$
$$\boxed{y = a\sin \frac{2\pi}{\lambda}(vt - x)}$$

This equation represents the equation of a progressive wave and gives the displacement of any particle whose distance x from fixed point A at any time is known. If the wave is travelling from right to left

$$\boxed{y = a\sin \frac{2\pi}{\lambda}(vt + x)}$$

It can be written in another form as, we know that $v = n\lambda$

$$y = a\sin 2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right)$$
$$= a\sin 2\pi\left(\frac{n\lambda t}{\lambda} - \frac{x}{\lambda}\right) = a\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$$
$$= a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$ where T – Time period – Time for one complete oscillation
$$= a\sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$ $\boxed{n = \frac{1}{T}}$

By putting, $\omega = \frac{2\pi}{T}$, $\frac{2\pi}{\lambda} = K$ (angular wave no.)

$$\boxed{y = a\sin(\omega t - Kx)}$$

This is another form of equation of a progressive wave.

Que6: Find differential form of wave equation.

Differential form of wave equation

General form of a progressive wave equation is

$$y = a\sin \frac{2\pi}{\lambda}(vt - x) \quad \text{--- (i)}$$

Differentiating equation (i) w.r.t. x

$$\frac{dy}{dx} = a \cdot \frac{2\pi}{\lambda}\cos \frac{2\pi}{\lambda}(vt - x)\cdot(-1) = -a\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)$$

$$\frac{d^2y}{dx^2} = +a\left(\frac{2\pi}{\lambda}\right)^2 \sin\frac{2\pi}{\lambda}(vt - x)\cdot(-1) = -a\frac{4\pi^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{--- (ii)}$$

Similarly, Differentiating equation (i) w.r.t. t:

$$\frac{dy}{dt} = a\left(\frac{2\pi}{\lambda}\right)\cdot\cos\frac{2\pi}{\lambda}(vt - x)\cdot v$$

$$\frac{d^2y}{dt^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 \sin\frac{2\pi}{\lambda}(vt - x)\cdot v^2 = -a\frac{4\pi^2 v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{--- (iii)}$$

Comparing equation (ii) & (iii)

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$

This is a differential form of wave equation. Any equation of this form always represents a wave motion and square root of coefficient $\frac{d^2y}{dx^2}$ represents wave velocity.

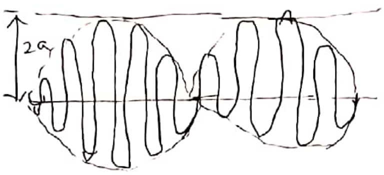
Que 7: Find the relation between Wave Velocity and Group velocity.

Ans: Let us Consider a group of waves Consisting of only two Components of equal amplitudes and having frequencies ω_1 and ω_2 differing by a small amount and represented by equations

$$y_1 = a\cos(\omega_1 t - k_1 x)$$
$$y_2 = a\cos(\omega_2 t - k_2 x)$$

where $\frac{\omega_1}{k_1} = V_1$ and $\frac{\omega_2}{k_2} = V_2$ their respective velocities and $\frac{\omega_1}{k_1} \neq \frac{\omega_2}{k_2}$

The resultant amplitude



$$Y = y_1 + y_2$$
$$= a\cos(\omega_1 t - k_1 x) + a\cos(\omega_2 t - k_2 x)$$
$$= 2a\cos\left[\frac{\omega_1+\omega_2}{2}t - \frac{(k_1+k_2)}{2}x\right]\cos\left[\frac{(\omega_1-\omega_2)}{2}t - \frac{(k_1-k_2)}{2}x\right]$$

$$\therefore y = 2a\cos(\omega t - kx)\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right) \quad \text{--- (1)}$$

where, $\omega = \frac{\omega_1+\omega_2}{2}$ and $\frac{k_1+k_2}{2} = k$, $\Delta\omega = \omega_1 - \omega_2$ and $\Delta k = (k_1 - k_2)$

The above equation (1) shows that the resultant wave has two parts.

(a) A wave of frequency ω, propagation Constant k and velocity $v = \frac{\omega}{k}$

(b) A second wave of frequency $\frac{\Delta\omega}{2}$, propagation Constant $\frac{\Delta k}{2}$ and velocity $V_g = \frac{\Delta\omega}{\Delta k}$

The first type of wave has maximum amplitude 2a. It is modulated in space and time by the second wave, which consists of a group of waves of the first type and is a very slowly varying envelope of frequency $\frac{\Delta\omega}{2}$ and propagation Constant $\frac{\Delta k}{2}$. The modulated pattern moves with a velocity $U_g = \frac{\Delta\omega}{\Delta k}$, known as group velocity.

Relation between wave velocity & Group velocity: Suppose the two frequency Components have different phase velocities $\frac{\omega_1}{k_1}$ and $\frac{\omega_2}{k_2}$, then the group velocity $U_g = \frac{\omega_1-\omega_2}{k_1-k_2} = \frac{\Delta\omega}{\Delta k}$ will be different from each of the phase velocities. In such a Case the superposition of two waves will no longer remain constant and hence group profile. Such a medium in which the phase velocity depends upon frequency is called dispersive medium. The variation of ω as a function of k represents a dispersion relation.

If the group of waves Consists of a no. of Component velocities very close to each other, we Can write

$$\frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk} = U_g$$

If the phase velocity is represented by U, then $U = \frac{\omega}{k}$

$$\text{or } \omega = kU$$

$$\frac{d\omega}{dk} = U + k\frac{dU}{dk} \quad \text{--- (II)}$$

$\because U = \frac{\omega}{k}$

$k = \frac{\omega}{U} = \frac{2\pi\nu}{U} = \frac{2\pi}{\lambda}$

$\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2} = -\frac{k}{\lambda}$

$\therefore \frac{k}{dk} = -\frac{\lambda}{d\lambda}$

So from equation (2)

$$\boxed{V_g = U - \lambda\cdot\frac{dU}{d\lambda}}$$

This is relation between group velocity and wave velocity

For dispersive medium, $\frac{dU}{d\lambda}$ is positive, $V_g < U$ Example. Electromagnetic waves in glass, water etc.

For non-dispersive medium, $\frac{dU}{d\lambda} = 0$. $V_g = U$ Example Electromagnetic waves in