SEMESTER VI Core Paper (13) (1)

Que 1: What do you mean by 'Electromagnetic waves'?

Ans: It is experimental fact that a stationary charge produces only electrostatic field; while a moving charge produces electric current which gives rise to magnetic field. According to Faraday's Laws of electromagnetic induction, a time varying magnetic field behaves like a source of electric field and according to Maxwell's modification of Ampere's law, a changing electric field gives rise to a magnetic field. This means that when either of the electric and magnetic fields changes with time the other field is induced in space. This leads to the generation of electromagnetic disturbance comprising of time varying electric and magnetic fields.

Thus we can say that an accelerated and decelerated charge, which produces time varying fields, gives rise to electromagnetic disturbances which can be propagated through space in the form of waves called Electromagnetic waves. Wireless waves, X-rays, γ-rays etc. are electromagnetic radiations and propagated through free space with velocity of $c = 3\times10^8$ m/sec.

Que 2: What do you mean by Conservation of charge. Define 'Equation of Continuity'.

Ans: According to principle of Conservation of charge, "the net amount of charge in an isolated system remains constant".

But if the charge density is a function of time, then the principle of conservation of charge may be stated as follows: "If the net charge crossing a surface bounding a closed volume is not zero, then the charge density within the volume must change with time in such a manner that the time rate of increase of charge within the volume equals to the net flow of charge into the volume". This statement of conservation of charge in a medium is known as Equation of Continuity which can be expressed as

$$\boxed{\mathrm{div}\, J + \frac{\partial \rho}{\partial t} = 0}$$

, where $J \rightarrow$ Current density (Current per unit area) amp/meter^2

ρ – charge density in Coulomb/meter^3

If there is no accumulation of charge or stationary current $\frac{\partial \rho}{\partial t} = 0$

$$\boxed{\nabla J = \mathrm{div}\, J = 0}$$

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Q.no 3: Write four Maxwell's field equation and derive them.

Ans: The four Maxwell's field equations can be written as;

(1) $\nabla D = \rho$

(2) $\nabla B = 0$

(3) $\nabla \times E = -\dfrac{\partial B}{\partial t}$

(4) $\nabla \times H = J + \dfrac{\partial D}{\partial t}$

where, D = Electric displacement vector
ρ = Charge density
B = Magnetic Induction
E = Electric field intensity
H = Magnetic field intensity
J = Current density

(1) Derivation of 1st equation $\nabla D = \rho$

Let us consider a surface S bounding by a volume V in a dielectric medium. In a dielectric medium total charge is sum of total charge ρ and polarisation charge. If ρ and ρ_p are the charge densities of free charge at a point in small volume element dV, then Gauss law can be expressed as

$$\int_S E \cdot ds = \frac{1}{\epsilon_0} \int (\rho + \rho_p) \, dV$$

But, Polarisation charge density $\rho_p = -\operatorname{div} P$, therefore

$$\int_S \epsilon_0 E \cdot ds = \int_V \rho \cdot dV - \int_V \operatorname{div} P \cdot dV \quad \text{--- (1)}$$

But from Gauss's divergence theorem.

$$\int_S \epsilon_0 E \cdot ds = \int_V \operatorname{div}(\epsilon_0 E) \, dV$$

Therefore, equation (1) gives

$$\int_V \operatorname{div}(\epsilon_0 E) \, dV = \int_V \rho \cdot dV - \int_V \operatorname{div} P \cdot dV$$

$$\int_V \operatorname{div}(\epsilon_0 E + P) \, dV = \int_V \rho \cdot dV$$

$$\int_V \operatorname{div} D \cdot dV = \int_V \rho \cdot dV \qquad] \because D = \epsilon_0 E + P$$

$$\therefore \int_V \operatorname{div}(D - \rho) \, dV = 0$$

As, volume is arbitrary $\operatorname{div} D - \rho = 0$ or. $\operatorname{div} D = \rho$

$$\boxed{\nabla D = \rho}$$

This is first Maxwell's Equation

(2) Derivation of 2nd Equation $\boxed{\nabla B = 0}$

magnetic lines of force generally are either closed curves or go off to infinity. Consequently the magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it. It means flux of magnetic Induction B across any closed surface is

Using Gauss's divergence Theorem

$$\int_S B \cdot ds = \int_V \nabla B \cdot dV$$

So, $$\int_S B \cdot ds = \int_V \nabla B \cdot dV = 0$$

As the Surface bounding the volume is arbitrary, therefore this equation holds only if the integrand vanishes, div $B = 0$

$$\boxed{\nabla B = 0}$$ This is 2nd Maxwell's Equation.

(3) Maxwell's Third Equation $$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

According to Faraday's Law of electromagnetic induction it is known that e.m.f. induced in a closed loop is defined as negative rate of change of magnetic flux, i.e. $$\boxed{e = -\frac{d\phi}{dt}}$$

But magnetic flux $\phi = \int_S B \cdot ds$, where S is any surface having loop as boundary,

$$e = -\frac{d}{dt}\int_S B \cdot ds$$

$$= -\int_S \frac{\partial B}{\partial t} ds \quad \text{(II)}$$ ∵ Surface is fixed in space, so only B changes with time

But emf 'e' Can be Computed by Calculating the work done in Carrying a unit charge round the closed loop C. Thus if E is the electric field intensity at a small element dl of the loop, we have $e = \int_C E \cdot dl$ (III)

Comparing equation (II) & (III)

$$\int_C E \cdot dl = -\int_S \frac{\partial B}{\partial t} \cdot ds \quad \text{--- (IV)}$$

Using Stoke's theorem change line integral to surface integral

$$\int_S \text{Curl } E \cdot ds = -\int \frac{\partial B}{\partial t} \cdot ds$$

$$\int_S \left(\text{Curl } E + \frac{\partial B}{\partial t}\right) \cdot ds = 0$$

If the Surface is arbitrary, $$\text{Curl } E + \frac{\partial B}{\partial t} = 0$$

$$\text{Curl } E = -\frac{\partial B}{\partial t}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

This is the third Maxwell's Equation

(4) Maxwell's Fourth Equation

From Ampere Circuital Law

$$\oint H \cdot dl = \int_S J \cdot ds \quad \text{—— (V)}$$

Where $I = \int_S J \cdot ds$ is the current enclosed by path and $\oint$ represents the integral over the closed path around surface S.

But from Stoke's Theorem

$$\oint_C H \cdot dl = \int_S \text{Curl } H \cdot ds$$

Using this equation the equation (V) gives

$$\int_S \text{Curl } H \cdot ds = \int_S J \cdot ds$$

$$\int_S (\text{Curl } H - J)\, ds = 0$$

As the surface is arbitrary, Curl $H - J = 0$ or Curl $H = J$ (VI)

This equation is invalid for time varying fields. Since div of curl of any vector is zero, therefore div Curl $H = 0$,

So, equation (VI) div Curl H = div $J = 0$

div $J = 0$ doesnot obey the Continuity equation representing the Conservation of charge

$$\text{div } J = -\frac{\partial \rho}{\partial t}.$$

Thus we conclude that Ampere's equation (V) is valid only for steady state conditions for which div $J = 0$ and is insufficient for time varying fields. Hence Maxwell's modified Ampere's Law to include time varying fields by adding another current density term J_d to J, so that equation (VI) would reduce to

$$\text{Curl } H = J + J_d \quad \text{(VII)}$$

, so that continuity equation would be satisfied

Taking divergence of equation (VII) both side, we have

$$\text{div. Curl } H = \text{div}(J + J_d) = 0$$

$$\text{div } J + \text{div. } J_d = 0$$

$$\text{div } J_d = -\text{div } J = \frac{\partial \rho}{\partial t}$$

But from differential form of Gauss's theorem, div $D = \rho$

$$\text{div } J_d = \frac{\partial}{\partial t}(\text{div } D) = \text{div} \frac{\partial D}{\partial t}$$

$$\therefore J_d = \frac{\partial D}{\partial t}$$

Thus modified form of Ampere's Law

$$\text{Curl } H = J + \frac{\partial D}{\partial t}, \text{ or } \boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

This is maxwell's fourth equation.