

Statistical Mechanics

Ques: What do you mean by a phase space?

Ans. In classical mechanics the state of a particle is completely specified by three position coordinates x, y, z and three momentum coordinates P_x, P_y, P_z . Now, if we imagine a six dimensional space in which $dx, dy, dz, dP_x, dP_y, dP_z$ is an element of volume and the position of a point particle in this space will be described by a set of six coordinates x, y, z, P_x, P_y, P_z . This six dimensional space for single particle is term as phase space or μ -space.

If the system contains a large no. of particles such that the state of system is represented by f independent position coordinates $q_1, q_2, q_3, \dots, q_f$ and f independent momentum coordinates $p_1, p_2, p_3, \dots, p_f$, then mathematically these $2f$ combined position-momentum coordinates may be allowed to define $2f$ dimensional space in which the configuration of the system is represented by $2f$ coordinates $q_1, q_2, q_3, \dots, q_f; p_1, p_2, p_3, \dots, p_f$. This $2f$ dimensional space is called phase space.

The instantaneous state of a particle in phase space is represented by a point called phase point. The no. of phase point per unit volume is called phase density and an element of volume in phase space is called a cell. A cell may contain a ~~small~~ large no. of these points.

Teachers Signature.....

Que: What do you mean by a Micro and Macro states?

Ans: Macrostate system

A macrostate system is the arrangement of particles in different cells of a phase space when the particles are indistinguishable.

Microstate system

A microstate system is the arrangement of particles in different cells of a phase space when the particles are distinguishable.

The no. of microstates are equal or greater than macrostates in other hand we can say that many different microstates may correspond to same macrostate.

Que 3: Four distinguishable coins are tossed for a large no. of times. Write down the different microstates which may be observed and the macrostate into which they would fall. Write down the probability of the most probable macrostate.

Ans: Let us denote the coins by a, b, c, d. If they are tossed for a large no. of times, we shall have the no. of macrostates and microstates are as shown in the table 'A'

Table-A

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Microstates		Macrostate
Coin head up	Coin head tail up	$n_1 = \text{no. of head up}$ $n_2 = \text{no. of tail up}$
abcd	—	$n_1 = 4, n_2 = 0$
abc	d	$n_1 = 3, n_2 = 1$
abd	c	$n_1 = 3, n_2 = 1$
acd	b	$n_1 = 3, n_2 = 1$
ab	cd	$n_1 = 2, n_2 = 2$
ac	bd	$n_1 = 2, n_2 = 2$
bc	ad	$n_1 = 2, n_2 = 2$
bd	ac	$n_1 = 2, n_2 = 2$
cd	ab	$n_1 = 2, n_2 = 2$
a	bcd	$n_1 = 1, n_2 = 3$
b	acd	$n_1 = 1, n_2 = 3$
c	abd	$n_1 = 1, n_2 = 3$
d	abc	$n_1 = 1, n_2 = 3$
—	abcd	$n_1 = 0, n_2 = 4$

There is general formula for arranging n particles having two (2) options

The no. of macrostates = $n+1$

So, for above case no. of macrostates = $4+1 = 5$

The no. of microstates = $2^n = 2^4 = 16$

The most probable macrostate $n_1 = 2, n_2 = 2$

whose probability = $\frac{6}{16} = \frac{3}{8}$

Que 4: What is Elementary Concept of a Ensemble.

Ans: A system is defined as a collection of a no. of particles. An ensemble is defined as a collection of large no. of macroscopically identical, but essentially independent systems. By the term macroscopically identical we mean that each of the systems constituting an ensemble satisfies the same macroscopic conditions (i.e. volume, energy, pressure, total no. of particles etc.). By the term independent systems we mean that the systems constituting an ensemble are mutually non-interacting. In an ensemble the systems play the same role as the non-interacting molecules do in a gas. The macroscopic identity of the systems constituting an ensemble may be achieved by choosing the same values of some set of macroscopic parameters which uniquely determine the equilibrium state of the system.